

An improved double sampling s chart

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Double sampling (DS) s charts are designed to allow quick detection of a small shift in process standard deviations and to provide a quick response in an agile manufacturing environment. However, current developed DS s charts assume that the sample standard deviations follow a normal distribution. Although valid for relatively large sample sizes, this assumption has limited the application of DS s charts to the monitoring of manufacturing processes with relatively small sample sizes. An improved DS s chart is developed without the normality assumption of the sample standard deviations. The design of the improved DS s chart is formulated as a statistical design optimization problem and solved with a genetic algorithm. The efficiency of the improved DS s charts is compared with that of the DS s charts developed in previous research and with that of the traditional s charts.

1. Introduction

In any production process, regardless of how well designed or carefully maintained, a certain amount of inherent or natural variability or ‘background noise’ is always present. Variability in key quality characteristics usually arises from three sources: improperly adjusted machines, operator errors and/or defective raw materials. Such variability usually represents an unacceptable level of process performance. We refer to these sources of variability that are not a part of the chance cause pattern as ‘assignable causes’. Once a process is in a state of statistical control, it keeps producing acceptable products for relatively long periods. However, occasionally assignable causes will occur, seemingly at random, resulting in a ‘shift’ to an out-of-control state where a larger proportion of the process output does not conform to the requirements. A major objective of statistical quality control is to detect quickly the occurrence of assignable causes or process shifts so that investigation of the process and corrective action may be undertaken before a large number of non-conforming units are manufactured. The control chart is an on-line process control technique widely used for this purpose.

As today’s manufacturing firms are moving towards agile manufacturing, quick and economic on-line statistical process control solutions are in high demand. Double sampling (DS) X-bar control charts present as a good alternative for statistical process control in an agile manufacturing environment. The efficiency of the DS X-bar control charts for detecting changes in process mean has been compared with other types of control charts such as Shewhart, variable sampling intervals (VSI), cumulative-sum (CUSUM) and exponentially weighted moving averages (EWMA) by Daudin (1992). When being compared with a fixed sample

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size procedure, the DS procedure was chosen such that its average sample size when the process is in control is equal to the fixed sample size. The results of the comparison reported in Daudin (1992) are summarized below.

In comparing a DS control chart with VSI, the average time to signal (ATS) was used. ATS is defined as the expected value of the time between the start of the process and the time when the chart signals. The results of the comparison showed that when the time required in collecting and measuring the sample is negligible, the DS scheme is more efficient than VSI.

When being compared with a classic Shewhart chart, a DS chart showed better detection of smaller to moderate shifts. Detection of large shifts is better accomplished with the Shewhart chart. Another advantage of using a DS chart over the Shewhart chart is the dramatic decrease in average sample size. When the process is in control, this decrease is nearly 50%. Based on the results of the comparison, Daudin (1992) concluded that the DS X-bar chart might be substituted for the Shewhart X-bar chart whenever the improvement in efficiency outweighs the administration/trouble costs.

The comparison with the combined Shewhart CUSUM chart showed that the two-sided combined Shewhart CUSUM schemes are better for detecting small shifts, and the DS chart is better for detecting large shifts. The efficiency of the DS chart is close to that of the two-sided combined Shewhart CUSUM schemes with $k = 1$, but very different from the two-sided combined Shewhart CUSUM schemes with $k = 0.25$. Here, k is the reference value (or the allowance, or the slack value), and is often chosen as about halfway between the target value for the mean and the out-of-control value of the mean that we are interested to detect quickly (Montgomery 2001). Therefore, the DS chart may be appropriate when greater efficiency is required for small shifts than with the Shewhart chart, and protection against large shifts is also important. However, when large shifts are very unusual, the CUSUM or combined Shewhart CUSUM schemes are best.

In comparison with the EWMA scheme, the DS charts were designed to match average run lengths (ARL) with corresponding EWMA charts when the process is in control. When $\lambda = 0.75$, the DS chart has a greater efficiency than the EWMA chart. However, when $\lambda = 0.5$ or 0.25 , the EWMA chart is better for detecting small shifts, and the DS chart is better for detecting large shifts. Here $0 \leq \lambda \leq 1$ is a weighting factor, which is a smoothing constant in exponentially weighted moving average expression. As λ decreases, the weight on previous history $(1 - \lambda)$ increases (Grant and Leavenworth 1996).

The DS X-bar control charts assume that the process standard deviation remains unchanged throughout the whole course of the process control. Therefore, the DS X-bar charts alone are not sufficient to monitor the process as the process standard deviation could be shifted while the mean of the process stays the same. A change in process standard deviation with a fixed process mean could lead to a higher proportion of non-conforming parts from the process.

Motivated by the efficiency of the DS X-bar charts in detecting a small shift in the process means, He and Grigoryan (2002) developed a DS s chart by applying the same principle of the DS X-bar charts. In constructing the DS s chart, they assumed that the sample standard deviations follow a normal distribution. Although valid for relatively large sample sizes, this assumption has limited the application of the DS s charts for monitoring the manufacturing processes with relatively small sample sizes. In this paper, an improved DS s chart is developed without the normality

assumption of the sample standard deviations. The design of the improved DS s chart is formulated as a statistical design optimization problem and solved with a genetic algorithm. The efficiency of the improved DS s charts is compared with that of the DS s charts developed in previous research and that of the traditional s charts. The current paper generalizes the design procedure of the DS s chart described by the authors in the previous work and suggests DS s chart construction model based on the real distribution of the sample statistics when samples are taken from a normal distribution, which allows to overcome the limitations of the previous research. To ease the explanation, the DS s chart developed in He and Grigoryan (2002) is referred to as the DS s chart with normality assumption throughout the paper.

The remainder of the paper is organized as follows. In Section 2, background information on traditional and other s charts is provided. Section 3 is devoted to the presentation of the improved DS s chart. In Section 4, the use of a genetic algorithm for solving the statistical design optimization problem of the improved DS s chart is explained. In Section 5, the efficiency of the improved DS s chart is compared with that of the DS s chart with normality assumption and that of the traditional s charts. The results of the comparison are presented and discussed. Finally, Section 6 concludes the paper.

2. Background on traditional and other s charts

Let s be a sample standard deviation and σ be a process standard deviation. If samples are drawn from a population with a normal probability distribution, then the mean of s is computed as $c_4\sigma$. It is also known that, as sample size n increases, the distribution of s becomes increasingly closer to a symmetrical distribution (Grant and Leavenworth 1996). Theoretical knowledge of the distribution of s in samples from a normal universe is the basis for 3-sigma limits on the control charts for s . The sample standard deviation s is not an unbiased estimator of σ . If the underlying distribution is normal, then s actually estimates $c_4\sigma$, where c_4 is a constant that depends on the sample size n (Montgomery 2001). Furthermore, the standard deviation of s is $\sigma\sqrt{1-c_4^2}$. This information was used to establish a traditional control chart on s . For the case with known process standard deviation σ , since $E(s) = c_4\sigma$, the centerline for the chart is $c_4\sigma$. The upper control limits (UCL $_s$) and lower control limits (LCL $_s$) for a s chart can be computed as follows:

$$UCL_s = c_4\sigma + 3\sigma\sqrt{1-c_4^2}$$

$$LCL_s = c_4\sigma - 3\sigma\sqrt{1-c_4^2},$$

where

$$c_4 = \left(\sqrt{\frac{2}{n-1}} \right) \frac{\Gamma(n/2)}{\Gamma[(n-1)/2]},$$

where $\Gamma(\cdot)$ is the Gamma function.

In addition to the traditional s charts, researchers have developed other alternative s charts over the years. These include the warning line control scheme by Page (1963), the EWMA chart based on $y = \ln(s^2)$ of the sample variance for monitoring a process standard deviation by Crowder and Hamilton (1992), and the one-stage s chart by Davis (1999).

3. Improved DS *s* chart

In this section, the improved DS *s* chart is presented. It follows the same procedure as the DS *s* chart with normality assumption. Two successive samples are taken without any intervening time, which allow us to consider that the samples are coming from the same probability distribution. This could be achieved by collecting a master sample of $n_1 + n_2$ units all at the same time. First, analyse the n_1 units in the first sample, and then decide whether to analyse the remaining units in the second sample. Figure 1 shows the graphical representation of the DS *s* chart. The control limits of the chart are shown in the number of the standard deviations of the estimated sample standard deviation. All the steps involved in the DS *s* chart could be summarized into a simple procedure.

For the sake of explanation, the DS *s* control procedure developed in He and Grigoryan (2002) is represented here. Before the DS *s* chart procedure is presented, define:

$$c_{41} = \left(\sqrt{\frac{2}{n_1 - 1}} \right) \frac{\Gamma(n_1/2)}{\Gamma[(n_1 - 1)/2]}, \text{ coefficient for sample size } n_1;$$

$$c_4 = \left(\sqrt{\frac{2}{n_1 + n_2 - 2}} \right) \frac{\Gamma[(n_1 + n_2 - 1)/2]}{\Gamma[(n_1 + n_2 - 2)/2]}, \text{ coefficient for sample size } n_1 + n_2.$$

(See the derivation of c_4 in appendix A.)

3.1. *DS s* chart procedure

- (1) Take an initial sample of size n_1 . Calculate the standard deviation of the sample, s_1 .

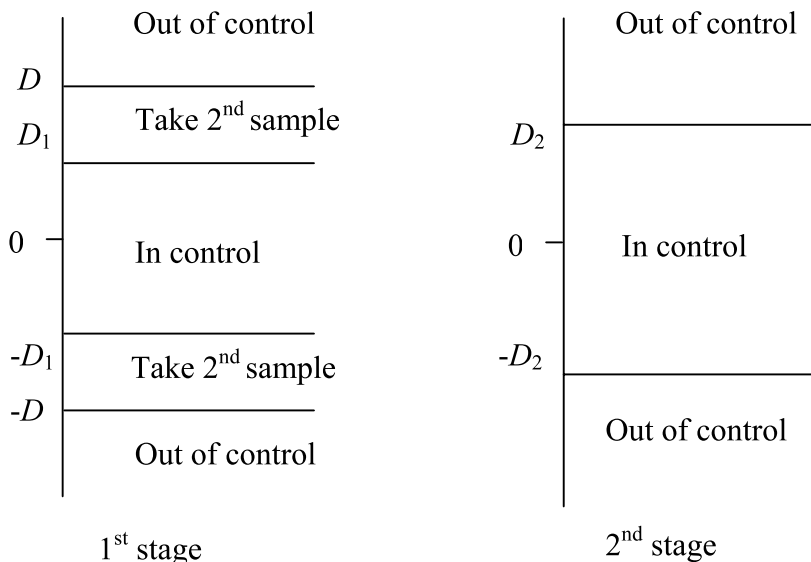


Figure 1. Graphic view of the DS *s* chart.

- (2) If $s_1 - c_{41}\sigma/\sigma\sqrt{1 - c_{41}^2}$ lies in the range $[-D_1, D_1]$, the process is considered to be under control.
- (3) If $s_1 - c_{41}\sigma/\sigma\sqrt{1 - c_{41}^2}$ lies in the range $(-\infty, -D]$ and $[D, +\infty)$, the process is considered to be out of control.
- (4) If $s_1 - c_{41}\sigma/\sigma\sqrt{1 - c_{41}^2}$ lies in the intervals $[-D, -D_1]$ and $[D_1, D]$, take a second sample of size n_2 and calculate the total sample standard deviation

$$s_{12} = \frac{\sqrt{\sum_{i=1}^2 (n_i - 1)s_i^2}}{\sqrt{\sum_{i=1}^2 n_i - 2}} = \sqrt{\frac{(n_1 - 1)s_1^2 + (n_2 - 1)s_2^2}{n_1 + n_2 - 2}}$$

- (5) If $s_{12} - c_4\sigma/\sigma\sqrt{1 - c_4^2}$ lies in the interval $[-D_2, D_2]$, then the process is in control, else the process is out of control.

The statistical design of a DS *s* chart involves determining the values of the following five parameters:

- n_1 sample size of the first sample,
- n_2 sample size of the second sample,
- $D_1, -D_1$ limits on the first sample within which the process is said to be in control,
- $D, -D$ limits on the first sample beyond which the process is said to be out of control,
- $D_2, -D_2$ limits on the second sample within which the process is said to be in control.

In general, three design criteria are used for design of a control chart: statistical design, economic design, and heuristic design (Saniga 1991). As pointed out by Woodall (1986), the economic method is often not effective in producing charts that can quickly detect small shifts before substantial losses can occur. Also discussed in Montgomery (1980), one of the problems associated with the economic design is the difficulty in estimating the costs. For these reasons, the statistical design method is chosen in this paper for the design of the double sampling *s* charts. In this paper, the statistical design of the improved DS *s* chart is formulated as an optimization problem. Before presenting the optimization model, the following intervals are defined:

$$I_1 = [c_{41}\sigma - D_1\sigma\sqrt{1 - c_{41}^2}, c_{41}\sigma + D_1\sigma\sqrt{1 - c_{41}^2}]$$

$$I_2 = [c_{41}\sigma - D\sigma\sqrt{1 - c_{41}^2}, c_{41}\sigma - D_1\sigma\sqrt{1 - c_{41}^2}]$$

$$\cup [c_{41}\sigma + D_1\sigma\sqrt{1 - c_{41}^2}, c_{41}\sigma + D\sigma\sqrt{1 - c_{41}^2}]$$

$$I_3 = (-\infty, c_{41}\sigma - D\sigma\sqrt{1 - c_{41}^2}] \cup [c_{41}\sigma + D\sigma\sqrt{1 - c_{41}^2}, \infty)$$

$$I_4 = [c_4\sigma - D_2\sigma\sqrt{1 - c_4^2}, c_4\sigma + D_2\sigma\sqrt{1 - c_4^2}]$$

$$I_5 = (-\infty, c_4\sigma - D_2\sigma\sqrt{1 - c_4^2}] \cup [c_4\sigma + D_2\sigma\sqrt{1 - c_4^2}, +\infty).$$

Let σ_0^2 and σ_1^2 be the target and shifted process variance, respectively. Mathematically, the optimization model can be written as follows:

$$\text{Min}_{n_1, n_2, D, D_1, D_2} n_1 + n_2 P_r[s_1 \in I_2 | \sigma = \sigma_0] \quad (1)$$

subject to

$$P_r[\text{Out of Control} | \sigma = \sigma_0] \leq \alpha \quad (2)$$

$$P_r[\text{In Control} | \sigma = \sigma_1] \leq \beta. \quad (3)$$

The objective function (1) is to minimize the average sample size when the process is in control. The reason for choosing objective function (1) is because the process is supposed to be in control most of the time and minimizing the sample size and hence the process monitoring costs is one of the goals in agile manufacturing. Constraint (2) ensures that the probability of making a false alarm is not greater than α (Type I error) and constraint (3) ensures that the probability of failure to detect a shift in process standard deviation is not greater than β (Type II error). In addition to constraints (2) and (3), lower and upper bounds are imposed on D , D_1 and D_2 . The values of the lower and upper bounds are set up as suggested by Daudin (1992) for practical implementation of DS charts. Daudin (1992) recommended that D must be higher than the classical values 3 or 3.09. A good choice is $D = 4$ or 5. In addition, integer constraints are imposed on n_1 and n_2 . In order to solve optimization models (1–3), the probabilities in constraints (2) and (3) should be defined in terms of the decision variables n_1 , n_2 , D , D_1 , and D_2 . Therefore, the derivation of probabilities:

$P_r[\text{Out of Control} | \sigma = \sigma_0]$ and $P_r[\text{In Control} | \sigma = \sigma_1]$ is provided next.

Let P_{a1} and P_{a2} be the probabilities that the process is in control at the first and second stage, respectively. Then, the probability that the process is in control can be computed as $P_a = P_{a1} + P_{a2}$. Consequently, the probability that the process is out of control is equal to: $1 - P_a$.

To compute the probability $P_r[\text{Out of Control} | \sigma = \sigma_0]$ in constraint (2) conveniently, some transformations are needed. It is known that if the universe is normal then the sample variance follows a χ^2 distribution. In computing probabilities, it makes no difference whether one works with the variance or the standard deviation, which is the square root of the variance, for the probability that s^2 will exceed a specified value s_1^2 is the same as the probability that s will exceed s_1 (Duncan 1986). Therefore, to calculate the probabilities of the sample standard deviations falling into a specified interval, the standard deviations of the first and second sample were transformed into following χ^2 random variables:

$$\begin{aligned}
 x_1 &= \frac{(n_1 - 1)s_1^2}{\sigma_0^2}, \text{ with } n_1 - 1 \text{ degrees of freedom} \\
 x_2 &= \frac{(n_2 - 1)s_2^2}{\sigma_0^2}, \text{ with } n_2 - 1 \text{ degrees of freedom} \\
 x_3 &= \frac{(n_1 - 1)s_1^2}{\sigma_1^2}, \text{ with } n_1 - 1 \text{ degrees of freedom} \\
 x_4 &= \frac{(n_2 - 1)s_2^2}{\sigma_1^2}, \text{ with } n_2 - 1 \text{ degrees of freedom.}
 \end{aligned}$$

When there is no shift in process standard deviation, the probability that process is in control at the first stage is computed as:

$$\begin{aligned}
 P_{a1}(\sigma = \sigma_0) &= \Pr[s_1 \in I_1, \sigma = \sigma_0] \\
 &= \int_{x_1 \in I_1^*} \frac{1}{2^{(n_1-1)/2} \Gamma(n_1 - 1/2)} x_1^{[(n_1-1/2)-1]} e^{-x_1/2} dx_1,
 \end{aligned}$$

where I_1^* can be written as follows:

$$I_1^* = \left[(n_1 - 1) \left(c_{41} - D_1 \sqrt{1 - c_{41}^2} \right)^2, (n_1 - 1) \left(c_{41} + D_1 \sqrt{1 - c_{41}^2} \right)^2 \right],$$

the transformation of interval I_1 into interval I_1^* is similar to that of I_2 into I_2^* , which is presented in appendix B.

The probability that the process is in control at the second stage when there is no shift in the process standard deviation can be derived as follows:

$$\begin{aligned}
 P_{a2}(\sigma = \sigma_0) &= \Pr[s_{12} \in I_4, \sigma = \sigma_0] \\
 &= \Pr[c_4\sigma_0 - D_2\sigma_0\sqrt{1 - c_4^2} \leq s_{12} \leq c_4\sigma_0 + D_2\sigma_0\sqrt{1 - c_4^2}] \\
 &= \Pr \left[c_4\sigma_0 - D_2\sigma_0\sqrt{1 - c_4^2} \leq \sqrt{\frac{(n_1 - 1)s_1^2 + (n_2 - 1)s_2^2}{n_1 + n_2 - 2}} \right] \\
 &\leq c_4\sigma_0 + D_2\sigma_0\sqrt{1 - c_4^2} \\
 &= \Pr \left[(c_4\sigma_0 - D_2\sigma_0\sqrt{1 - c_4^2})^2 \leq \frac{(n_1 - 1)s_1^2 + (n_2 - 1)s_2^2}{n_1 + n_2 - 2} \right] \\
 &\leq (c_4\sigma_0 + D_2\sigma_0\sqrt{1 - c_4^2})^2 \\
 &= \Pr \left[\sigma_0^2(c_4 - D_2\sqrt{1 - c_4^2})^2 \leq \frac{(n_1 - 1)s_1^2 + (n_2 - 1)s_2^2}{n_1 + n_2 - 2} \right] \\
 &\leq \sigma_0^2(c_4 + D_2\sqrt{1 - c_4^2})^2
 \end{aligned}$$

$$\begin{aligned}
 &= \Pr \left[(c_4 - D_2 \sqrt{1 - c_4^2})^2 \frac{(n_1 - 1)s_1^2/\sigma_0^2 + (n_2 - 1)s_2^2/\sigma_0^2}{n_1 + n_2 - 2} \right. \\
 &\quad \left. \leq (c_4 + D_2 \sqrt{1 - c_4^2})^2 \right].
 \end{aligned}$$

Let $r = n_1 + n_2 - 2$, then the conditional probability is computed as follows:

$$\Pr[s_{12} \in I_4|x_1] = \Pr[r(c_4 - D_2 \sqrt{1 - c_4^2})^2 - x_1 x_2 r(c_4 + D_2 \sqrt{1 - c_4^2})^2 - x_1].$$

Let

$$A = r(c_4 - D_2 \sqrt{1 - c_4^2})^2 - x_1$$

$$B = r(c_4 + D_2 \sqrt{1 - c_4^2})^2 - x_1$$

$$\begin{aligned}
 P_{a2}(\sigma = \sigma_0) &= \int_{x_1 \in I_2^*} \{\Pr[s_{12} \in I_4|x_1]\} f(x_1) dx_1 \\
 &= \int_{x_1 \in I_2^*} \left[\int_{x_2 \in [A, B]} \frac{x_2^{[(n_2-1)/2]-1} e^{-x_2/2}}{2^{(n_2-1)/2} \Gamma\left(\frac{n_2-1}{2}\right)} dx_2 \right] \frac{x_1^{[(n_1-1)/2]-1} e^{-x_1/2}}{2^{(n_1-1)/2} \Gamma\left(\frac{n_1-1}{2}\right)} dx_1,
 \end{aligned}$$

where I_2^* can be written as follows. (See the derivation of I_2^* in appendix B.):

$$\begin{aligned}
 I_2^* &= [(n_1 - 1)(c_{41} - D_1 \sqrt{1 - c_{41}^2})^2, (n_1 - 1)(c_{41} - D_1 \sqrt{1 - c_{41}^2})^2] \\
 &\quad \cup [(n_1 - 1)(c_{41} + D_1 \sqrt{1 - c_{41}^2})^2, (n_1 - 1)(c_{41} + D_1 \sqrt{1 - c_{41}^2})^2].
 \end{aligned}$$

Thus, constraint (2) can be written as:

$$\begin{aligned}
 P_r[\text{Out of Control}|\sigma = \sigma_0] &\leq \alpha, \quad \text{i.e. } 1 - [P_{a1}(\sigma = \sigma_0) + P_{a2}(\sigma = \sigma_0)] \leq \alpha \\
 &1 - \left\{ \int_{x_1 \in I_1^*} \frac{1}{2^{(n_1-1)/2} \Gamma\left(\frac{n_1-1}{2}\right)} x_1^{[(n_1-1)/2]-1} e^{-x_1/2} dx_1 \right\} \\
 &- \int_{x_1 \in I_2^*} \left[\int_{x_2 \in [A, B]} \frac{x_2^{[(n_2-1)/2]-1} e^{-x_2/2}}{2^{(n_2-1)/2} \Gamma\left(\frac{n_2-1}{2}\right)} dx_2 \right] \frac{x_1^{[(n_1-1)/2]-1} e^{-x_1/2}}{2^{(n_1-1)/2} \Gamma\left(\frac{n_1-1}{2}\right)} dx_1 \leq \alpha
 \end{aligned}$$

Constraint (3) ensures that the probability of failure to detect a shift in the process standard deviation less than or equal to predefined acceptable value β :

$$[P_{a1}(\sigma = \sigma_1) + P_{a2}(\sigma = \sigma_1)] \leq \beta.$$

When there is a shift in the process standard deviation from σ_0 to σ_1 , then the probability that the process is in control at the first stage is equal to:

$$\begin{aligned}
 P_{a2}(\sigma = \sigma_1) &= \Pr[s_1 \in I_1, \sigma = \sigma_1], \\
 &= \int_{x_3 \in I_1^{**}} \frac{x_3^{[(n_1-1)/2]-1} e^{-x_3/2}}{2^{(n_1-1)/2} \Gamma\left(\frac{n_1-1}{2}\right)} dx_3
 \end{aligned}$$

where

$$I_1^{**} = \left[(n_1 - 1) \frac{\sigma_0^2}{\sigma_1^2} (c_{41} - D_1 \sqrt{1 - c_{41}^2})^2, \quad (n_1 - 1) \frac{\sigma_0^2}{\sigma_1^2} (c_{41} + D_1 \sqrt{1 - c_{41}^2})^2 \right],$$

the transformation of interval I_1 into interval I_1^{**} , is similar to that of I_2 into I_2^{**} , which is presented in appendix B.

When there is a shift in the process standard deviation from σ_0 to σ_1 , then the probability that the combined sample statistic falls within the control limits is computed as:

$$\begin{aligned}
 &\Pr \left[\frac{\sigma_0^2}{\sigma_1^2} (c_4 - D_2 \sqrt{1 - c_4^2})^2 \leq \frac{(n_1 - 1) \frac{s_1^2}{\sigma_1^2} + (n_2 - 1) \frac{s_2^2}{\sigma_1^2}}{n_1 + n_2 - 2} \leq \frac{\sigma_0^2}{\sigma_1^2} (c_4 + D_2 \sqrt{1 - c_4^2})^2 \right] \\
 &= \Pr \left[r \frac{\sigma_0^2}{\sigma_1^2} (c_4 - D_2 \sqrt{1 - c_4^2})^2 - x_3 \leq x_4 \leq r \frac{\sigma_0^2}{\sigma_1^2} (c_4 + D_2 \sqrt{1 - c_4^2})^2 - x_3 \right].
 \end{aligned}$$

Define:

$$\begin{aligned}
 A_1 &= r \frac{\sigma_0^2}{\sigma_1^2} (c_4 - D_2 \sqrt{1 - c_4^2})^2 - x_3 \\
 B_1 &= r \frac{\sigma_0^2}{\sigma_1^2} (c_4 + D_2 \sqrt{1 - c_4^2})^2 - x_3.
 \end{aligned}$$

Thus, the probability that random variable s_{12} will fall into interval I_4 and random variable s_1 falls to interval I_2 is computed as:

$$\begin{aligned}
 P_{a2}(\sigma = \sigma_1) &= \int_{x_3 \in I_2^{**}} \{ \Pr[s_{12} \in I_4 | x_3] \} f(x_3) dx_3 \\
 &= \int_{x_3 \in I_2^{**}} \left[\int_{x_4 \in [A_1, B_1]} \frac{x_4^{[(n_2-1)/2]-1} e^{-x_4/2}}{2^{(n_2-1)/2} \Gamma\left(\frac{n_2-1}{2}\right)} dx_4 \right] \frac{x_3^{[(n_1-1)/2]-1} e^{-x_3/2}}{2^{(n_1-1)/2} \Gamma\left(\frac{n_1-1}{2}\right)} dx_3.
 \end{aligned}$$

Constraint (3) can be written as:

$$P_r[\text{In Control} | \sigma = \sigma_1] \leq \beta$$

(for a given intended change in process standard deviation), i.e.

$$\int_{x_3 \in I_1^{**}} \frac{x_3^{(n_1-1/2)-1} e^{-x_3/2}}{2^{(n_1-1/2)} \Gamma\left(\frac{n_1-1}{2}\right)} dx_3$$

$$+ \int_{x_3 \in I_2^{**}} \left[\int_{x_4 \in [A_1, B_1]} \frac{x_4^{[(n_2-1)/2]-1} e^{-x_4/2}}{2^{(n_2-1)/2} \Gamma\left(\frac{n_2-1}{2}\right)} dx_4 \right] \frac{x_3^{[(n_1-1)/2]-1} e^{-x_3/2}}{2^{(n_1-1)/2} \Gamma\left(\frac{n_1-1}{2}\right)} dx_3 \leq \beta,$$

where I_2^{**} can be written as follows (see the derivation of I_2^{**} in appendix B):

$$I_2^{**} = \left[(n_1 - 1) \frac{\sigma_0^2}{\sigma_1^2} (c_{41} - D\sqrt{1 - c_{41}^2})^2, \quad (n_1 - 1) \frac{\sigma_0^2}{\sigma_1^2} (c_{41} - D_1\sqrt{1 - c_{41}^2})^2 \right]$$

$$\cup \left[(n_1 - 1) \frac{\sigma_0^2}{\sigma_1^2} (c_{41} + D_1\sqrt{1 - c_{41}^2})^2, \quad (n_1 - 1) \frac{\sigma_0^2}{\sigma_1^2} (c_{41} + D\sqrt{1 - c_{41}^2})^2 \right].$$

Then, the optimization model (1–3) can be rewritten as follows:

$$\text{Min}_{n_1, n_2, D, D_1, D_2} n_1 + n_2 \int_{x_1 \in I_2^*} \frac{x_1^{[(n_1-1)/2]-1} e^{-x_1/2}}{2^{(n_1-1)/2} \Gamma\left(\frac{n_1-1}{2}\right)} dx_1 \tag{4}$$

subject to

$$1 - \left\{ \int_{x_1 \in I_1^*} \frac{x_1^{(n_1-1/2)-1} e^{-x_1/2}}{2^{\frac{(n_1-1)}{2}} \Gamma\left(\frac{n_1-1}{2}\right)} dx_1 \right\}$$

$$- \int_{x_1 \in I_2^*} \left[\int_{x_2 \in [A, B]} \frac{x_2^{[(n_2-1)/2]-1} e^{-x_2/2}}{2^{(n_2-1)/2} \Gamma\left(\frac{n_2-1}{2}\right)} dx_2 \right] \frac{x_1^{[(n_1-1)/2]-1} e^{-x_1/2}}{2^{(n_1-1)/2} \Gamma\left(\frac{n_1-1}{2}\right)} dx_1 \leq \alpha \tag{5}$$

$$\int_{x_3 \in I_1^{**}} \frac{x_3^{[(n_1-1)/2]-1} e^{-x_3/2}}{2^{(n_1-1)/2} \Gamma\left(\frac{n_1-1}{2}\right)} dx_3$$

$$+ \int_{x_3 \in I_2^{**}} \left[\int_{x_4 \in [A_1, B_1]} \frac{x_4^{[(n_2-1)/2]-1} e^{-x_4/2}}{2^{(n_2-1)/2} \Gamma\left(\frac{n_2-1}{2}\right)} dx_4 \right] \frac{x_3^{[(n_1-1)/2]-1} e^{-x_3/2}}{2^{(n_1-1)/2} \Gamma\left(\frac{n_1-1}{2}\right)} dx_3 \leq \beta. \tag{6}$$

4. Solving the optimization problem using genetic algorithms (GAs)

As one can see that the minimization problem formulated by models (4–6) is characterized by mixed continuous-discrete variables, and discontinuous and non-convex solution space. Therefore, if standard non-linear programming techniques are used for solving this type of optimization problem they will be inefficient and computationally expensive. GAs are well suited for solving such problems, and in most cases find a global optimum solution with a high probability (Rao 1996).

Another motivation in using GAs to solve models (4–6) is that since the pioneer work by Holland (1975), GAs have been developed into a general and robust method for solving all kinds of optimization problems (e.g. Goldberg 1989, Potgieter and Stander 1998, Pham and Pham 1999, Vinterbo and Ohno-Machado 2000) and computing software for applications of GAs have been commercially available in the market. Thus, solving models (4–6) using a GA provides a practical way for real-time statistical process control implementation. The GAs have been used for the statistical design of double sampling and triple sampling \bar{X} -bar control charts (He *et al.* 2002) and the DS s charts with normality assumption (He and Grigoryan 2002).

The tool used for implementing the GAs for solving models (4–6) is a commercial software called Evolver (Palisade 1998). Evolver is used as an add-in program to the Microsoft Excel spreadsheet application. The optimization models (4–6) are set up in an Excel spreadsheet and solved by the GA in Evolver. The operation of the GA involves following steps: (1) create a random initial solution; (2) evaluate fitness, i.e. the objective function that minimizes the average sample size when the process is in control; (3) reproduction and mutation; and (4) generate new solutions.

The quality of the solutions generated by the GAs depends on the set-up of its parameters such as population size, crossover and mutation probability. During the implementation, the values of these parameters are set-up to obtain the best results.

Crossover probability determines how often crossover will be performed. If there is no crossover, an offspring (new solution) will be an exact copy of the parents (old solutions). If there is a crossover, an offspring (new solution) is made from parts of parents' chromosome. If crossover probability is 100%, then offspring are made by crossover. Crossover is made up in hope that new chromosomes will have good parts of old chromosomes and maybe the new chromosomes will be better. However, it is good to leave some part of the population to survive to the next generation.

Mutation probability determines how often parts of the chromosomes will be mutated. If there is no mutation, offspring will be taken after crossover (or copy) without any change. If mutation probability is 100%, the whole chromosome is changed. Mutation is made to prevent the search by genetic algorithm falling into local extremes, but it should not occur very often, because then the GA will in fact change to random search.

In our computational experiment, the population size is set to 1000. The crossover probability is set-up to 0.5 and mutation probability 0.06.

The integrations in (5) and (6) are computed with numerical integration using Simpson's rule (Gerald and Wheatley 1999).

5. Performance evaluation of the improved DS s chart

In order to evaluate the performance of the improved DS s chart, its efficiency (the average sample size when process is in control) was compared with that of the traditional one and the DS s chart with normality assumption presented in (He and Grigoryan 2002). For any control chart for variables, the average number of samples before a shift of certain magnitude is detected should be as small as possible. The average number of samples before the false alarm should be as large as possible. One way to evaluate the decisions about sample size and sampling frequency is through the average run length (ARL) of the control chart. Essentially, ARL is the average number of points that must be plotted before a point indicates an out-of-control condition (Montgomery 2001). For any Shewhart control chart, ARL can be

expressed as $ARL_0 = 1/\alpha$, for the in-control case and $ARL_1 = 1/(1 - \beta)$ for out-of-control case.

A comparison between the DS s charts and traditional s charts was first reported in He and Grigoryan (2002), they followed a procedure similar to that used by Daudin (1992) for comparing the DS X-bar charts with the Shewhart X-bar charts. In Daudin's procedure, the efficiency of a DS X-bar chart is compared with that of a corresponding Shewhart X-bar chart that is matched with the same ARL_0 and ARL_1 as the DS X-bar chart.

In the procedure, for a given pair of ARL_0 and ARL_1 points and a given shift in process standard deviation, a DS s chart and traditional s chart were constructed. Then, the efficiency of the two charts was compared. Here a shift in process standard deviation is expressed as a ratio of the shifted process standard deviation σ_1 to the normal process standard deviation σ_0 , $\lambda = \sigma_1/\sigma_0$.

In our computational experiment, the control charts were constructed for detecting shifts over a range of λ values from 0.6 to 5.0. The two ARL points used in the comparison were: $ARL_0 = 370.4$ ($\alpha = 0.0027$) and $ARL_1 = 1.222$ ($\beta = 0.1817$).

For constructing a corresponding improved DS s chart, the two ARL points (α and β) and the given σ_1 value (obtained from a given λ and known σ_0) were used to solve the statistical design optimization models (4–6) with the genetic algorithm.

Having constructed the charts, the efficiency of the charts is compared in terms of the average sample size when the process is in control. The results are provided in table 1. The numbers in the first column of the table indicate the sample sizes of the traditional s charts. The sample sizes at the first and the second stage of the improved DS s charts are provided in the next two columns. The numbers shown in D_1 , D and D_2 columns of the table represent the optimal values of the control limits of the improved DS s charts. The average sample sizes of the optimal statistical design of the DS s charts with normality assumption and the improved DS s charts are provided in the last two columns, respectively.

From table 1, one can observe that for the whole range of the investigated shifts ($0.6 \leq \lambda \leq 5$) in process standard deviation the improved DS s charts result in a significant reduction in average sample size in comparison with the traditional s charts. This result is expected and is consistent with the report by He and Grigoryan (2002) that the DS s charts produce a superior performance over the traditional s charts in terms of reducing the average sample size for relatively large sample sizes.

For the case $\lambda < 1$ the improved DS s charts behave as expected (i.e. larger average sample sizes are needed to detect smaller shifts and smaller average sample sizes are needed to detect larger shifts), although the detection of these types of shift in most cases is not of primary concern.

In comparison with the DS s charts with normality assumption, the improved DS s charts show a relatively smaller average sample size. The differences between the average sample sizes of the two DS s charts are not significant though when the shifts in the process standard deviations occur with $\lambda < 2.5$. This result suggests that for relatively large sample sizes the DS s charts with normality assumption provide a good approximation to the optimal solutions of the DS s charts. In this case, the use of the DS s charts with normality assumption offer a great advantage over that of the improved DS s charts in terms of the simplicity of the computational effort.

As λ increases beyond 2.5, the differences between the average sample sizes of the two DS s charts become significant. This result is also as expected since for relatively

<i>n</i>	DS <i>s</i> chart		$\lambda = \frac{\sigma_1}{\sigma_0}$	ARL ₀	ARL ₁	<i>D</i> ₁	<i>D</i>	<i>D</i> ₂	<i>E</i> (<i>N</i>) ^{<i>a</i>}	<i>E</i> (<i>N</i>) ^{<i>b</i>}
	<i>n</i> ₁	<i>n</i> ₂								
41	16	33	0.6	370.4	1.222	1.57	3.56	2.89	20.85	19.78
75	31	59	0.7	370.4	1.222	1.60	3.49	2.91	38.88	37.48
175	79	126	0.8	370.4	1.222	1.64	3.51	2.93	89.01	98.65
211	124	183	1.2	370.4	1.222	1.76	3.34	2.96	141.79	137.07
99	53	116	1.3	370.4	1.222	1.76	3.40	3.00	63.01	61.90
59	35	43	1.4	370.4	1.222	1.75	3.64	2.91	39.26	38.34
40	22	39	1.5	370.4	1.222	1.77	3.48	2.89	26.42	24.94
30	17	27	1.6	370.4	1.222	1.80	3.35	2.92	19.58	18.91
22	13	30	1.7	370.4	1.222	1.80	3.27	2.92	15.63	15.07
18	11	21	1.8	370.4	1.222	1.80	3.25	3.00	12.60	12.44
15	10	12	1.9	370.4	1.222	1.79	3.34	2.94	10.78	10.80
13	8	22	2.0	370.4	1.222	1.80	3.24	2.97	9.46	9.48
12	8	12	2.1	370.4	1.222	1.80	3.36	2.96	8.43	8.81
10	7	12	2.2	370.4	1.222	1.78	3.35	2.96	7.34	7.80
9	6	12	2.3	370.4	1.222	1.80	3.35	2.96	6.91	6.79
9	5	12	2.4	370.4	1.222	1.80	3.48	2.96	6.91	6.30
8	5	12	2.5	370.4	1.222	1.77	3.38	2.94	6.91	5.82
5	4	8	3.0	370.4	1.222	1.80	3.43	2.99	6.91	4.49
5	3	6	4.0	370.4	1.222	1.80	3.63	2.96	6.91	3.32
4	3	4	5.0	370.4	1.222	1.79	3.92	2.91	6.91	3.22

^{*a*} Average sample size of the DS *s* charts with normality assumption; ^{*b*} Average sample size of the improved DS *s* charts.

Table 1. Results of the comparisons.

small sample size the normality assumption of the sample standard deviations is no longer valid. For relatively large shifts, the traditional *s* charts even outperform the DS *s* charts with normality assumption. The explanation for such a behaviour is that when the sample size is relatively small ($n_1 \leq 5$), the DS *s* chart becomes one-sided. Indeed, when the chart is one-sided the area under the normal probability density function curve becomes less than 1. While sample size decreases the area under the normal distribution curve decreases accordingly. With the probability less than unity, the normal approximation cannot provide the satisfaction of constraints. The results confirm the validity of the improved DS *s* charts for process control with small sample sizes. The limitations of the DS *s* chart design procedure developed in the previous research work can be overcome by the generalized design procedure of the improved DS *s* charts developed in this paper.

6. Conclusions

DS X-bar control charts are designed to allow quick detection of a small shift of process mean and provide a quick response in an agile manufacturing environment. However, the DS X-bar control charts assume that the process standard deviation remains unchanged throughout the entire course of the statistical process control. Therefore, a DS *s* chart can be used as a complementary chart to monitor the process variation caused by changes in process standard deviation.

The DS s charts developed in the previous research assume that the sample standard deviations follow a normal distribution. Although valid for relatively large sample sizes, this assumption has limited the application of the DS s charts for monitoring the manufacturing processes with relatively small sample sizes. In this paper, an improved DS s chart is developed without the normality assumption of the sample standard deviations. The design of the improved DS s chart is formulated as a statistical design optimization problem and solved with a genetic algorithm. The efficiency of the improved DS s charts is compared with that of the DS s charts developed in previous research and with that of the traditional s charts.

The results of the comparison show that for the whole range of the investigated shifts ($0.6 \leq \lambda \leq 5$) in process standard deviation, the improved DS s charts result in a significant reduction in average sample size in comparison with the traditional s charts. In comparison with the DS s charts with normality assumption, the improved DS s charts show a relatively smaller average sample size. The results suggest that for relatively large sample sizes, the DS s charts with normality assumption provide a good approximation to the optimal solutions of the DS s charts. The results confirm the validity of the improved DS s charts for process control with relatively small sample sizes. The limitations of the DS s chart design procedure developed in the previous research work can be overcome by the generalized design procedure of the improved DS s charts developed here.

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Appendix A: Derivation of c_4

Implicitly, it is assumed that quality measurement follows a normal distribution with a mean μ and a standard deviation σ . As random variables $x = (n_1 - 1)s_1^2/\sigma^2$ and $y = (n_2 - 1)s_2^2/\sigma^2$ follow χ^2 distributions with degrees of freedom $n_1 - 1$ and $n_2 - 1$, respectively, the sum of two independent χ^2 's is a χ^2 distribution with degrees of freedom $(n_1 - 1) + (n_2 - 1) = n_1 + n_2 - 2$. It now follows that:

$$\begin{aligned} s_{12} &= \sqrt{\frac{(n_1 - 1)s_1^2 + (n_2 - 1)s_2^2}{n_1 + n_2 - 2}} = \sqrt{\frac{\sigma^2}{n_1 + n_2 - 2} \left[\frac{(n_1 - 1)s_1^2}{\sigma^2} + \frac{(n_2 - 1)s_2^2}{\sigma^2} \right]} \\ &= \sqrt{\frac{\sigma^2}{n_1 + n_2 - 2}} Y = \frac{\sigma}{\sqrt{n_1 + n_2 - 2}} Y^{1/2}, \end{aligned} \quad (\text{A } 1)$$

where Y has a χ^2 distribution with $n_1 + n_2 - 2$ degrees of freedom. Thus,

$$\begin{aligned} E(s) &= \mu_s = \frac{\sigma}{\sqrt{n_1 + n_2 - 2}} E[Y^{1/2}] \\ &= \frac{\sigma}{\sqrt{n_1 + n_2 - 2}} \int_0^\infty y^{1/2} \frac{y^{((n_1+n_2-1)/2)-1} e^{-y/2}}{\Gamma\left(\frac{n_1 + n_2 - 1}{2}\right) 2^{(n_1+n_2-1)/2}} dy \end{aligned}$$

$$\begin{aligned}
 &= \sigma \frac{\Gamma\left(\frac{n_1 + n_2 - 1}{2}\right) 2^{(n_1+n_2-1)/2}}{\sqrt{n_1 + n_2 - 2} \Gamma\left(\frac{n_1 + n_2 - 2}{2}\right) 2^{(n_1+n_2-1)/2}} \\
 &\quad \times \int_0^\infty y^{1/2} \frac{y^{((n_1+n_2-1)/2)-1} e^{-y/2}}{\Gamma\left(\frac{n_1 + n_2 - 1}{2}\right) 2^{(n_1+n_2-1)/2}} dy \\
 &= \left(\sqrt{\frac{2}{n_1 + n_2 - 2}}\right) \frac{\Gamma[(n_1 + n_2 - 1)/2]}{\Gamma[(n_1 + n_2 - 2)/2]} \sigma = c_4 \sigma. \tag{A 2}
 \end{aligned}$$

Appendix B: Derivation of intervals of I_2^* and I_2^{}**

Since s_1 is in interval I_2 and

$$\begin{aligned}
 I_2 &= [c_{41}\sigma_0 - D\sigma_0\sqrt{1 - c_{41}^2}, c_{41}\sigma_0 + D_1\sigma_0\sqrt{1 - c_{41}^2}] \\
 &\quad \cup [c_{41}\sigma_0 + D_1\sigma_0\sqrt{1 - c_{41}^2}, c_{41}\sigma_0 + D\sigma_0\sqrt{1 - c_{41}^2}],
 \end{aligned}$$

then:

$$c_{41}\sigma_0 - D\sigma_0\sqrt{1 - c_{41}^2} \leq s_1 \leq c_{41}\sigma_0 - D_1\sigma_0\sqrt{1 - c_{41}^2}$$

and

$$c_{41}\sigma_0 + D_1\sigma_0\sqrt{1 - c_{41}^2} \leq s_1 \leq c_{41}\sigma_0 + D\sigma_0\sqrt{1 - c_{41}^2}.$$

For the case when there is no shift in the process variability, i.e. $\lambda = 1$, by multiplying $(n_1 - 1)$ to and dividing by σ_0^2 , both sides of the inequality signs, we obtain

$$\frac{(n_1 - 1)(c_{41}\sigma_0 - D\sigma_0\sqrt{1 - c_{41}^2})^2}{\sigma_0^2} \leq \frac{s_1^2(n_1 - 1)}{\sigma_0^2} \leq \frac{(n_1 - 1)(c_{41}\sigma_0 - D_1\sigma_0\sqrt{1 - c_{41}^2})^2}{\sigma_0^2}$$

and

$$\frac{(n_1 - 1)(c_{41}\sigma_0 + D_1\sigma_0\sqrt{1 - c_{41}^2})^2}{\sigma_0^2} \leq \frac{s_1^2(n_1 - 1)}{\sigma_0^2} \leq \frac{(n_1 - 1)(c_{41}\sigma_0 + D\sigma_0\sqrt{1 - c_{41}^2})^2}{\sigma_0^2}.$$

Since x_1 is defined as $(n_1 - 1)s_1^2/\sigma_0^2$ then:

$$(n_1 - 1)(c_{41} - D\sqrt{1 - c_{41}^2})^2 \leq x_1 \leq (n_1 - 1)(c_{41} - D_1\sqrt{1 - c_{41}^2})^2$$

and

$$(n_1 - 1)(c_{41} + D_1\sqrt{1 - c_{41}^2})^2 \leq x_1 \leq (n_1 - 1)(c_{41} + D\sqrt{1 - c_{41}^2})^2.$$

Thus, interval I_2^* can be written as:

$$I_2^* = [(n_1 - 1)(c_{41} - D\sqrt{1 - c_{41}^2})^2, (n_1 - 1)(c_{41} - D_1\sqrt{1 - c_{41}^2})^2] \\ \cup [(n_1 - 1)(c_{41} + D_1\sqrt{1 - c_{41}^2})^2, (n_1 - 1)(c_{41} + D\sqrt{1 - c_{41}^2})^2].$$

When there is a shift in process standard deviation from σ_0 to σ_1 , then the interval I_2^{**} within which the random variable x should vary, could be obtained as follows:

We have:

$$c_{41}\sigma_0 - D\sigma_0\sqrt{1 - c_{41}^2} \leq s_1 \leq c_{41}\sigma_0 - D_1\sigma_0\sqrt{1 - c_{41}^2}$$

and

$$c_{41}\sigma_0 + D_1\sigma_0\sqrt{1 - c_{41}^2} \leq s_1 \leq c_{41}\sigma_0 + D\sigma_0\sqrt{1 - c_{41}^2}.$$

By multiplying $(n_1 - 1)$ to and dividing by σ_1^2 both sides of the inequality signs, we obtain:

$$\frac{(n_1 - 1)(c_{41}\sigma_0 - D\sigma_0\sqrt{1 - c_{41}^2})^2}{\sigma_1^2} \leq \frac{s_1^2(n_1 - 1)}{\sigma_1^2} \leq \frac{(n_1 - 1)(c_{41}\sigma_0 - D_1\sigma_0\sqrt{1 - c_{41}^2})^2}{\sigma_1^2},$$

and

$$\frac{(n_1 - 1)(c_{41}\sigma_0 + D_1\sigma_0\sqrt{1 - c_{41}^2})^2}{\sigma_1^2} \leq \frac{s_1^2(n_1 - 1)}{\sigma_1^2} \leq \frac{(n_1 - 1)(c_{41}\sigma_0 + D\sigma_0\sqrt{1 - c_{41}^2})^2}{\sigma_1^2}.$$

Since x_3 is defined as $(n_1 - 1)s_1^2/\sigma_1^2$ then:

$$(n_1 - 1)\frac{\sigma_0^2}{\sigma_1^2}(c_{41} - D\sqrt{1 - c_{41}^2})^2 \leq x_3 \leq (n_1 - 1)\frac{\sigma_0^2}{\sigma_1^2}(c_{41} - D_1\sqrt{1 - c_{41}^2})^2$$

and

$$(n_1 - 1)\frac{\sigma_0^2}{\sigma_1^2}(c_{41} + D_1\sqrt{1 - c_{41}^2})^2 \leq x_3 \leq (n_1 - 1)\frac{\sigma_0^2}{\sigma_1^2}(c_{41} + D\sqrt{1 - c_{41}^2})^2.$$

Therefore, I_2^{**} can be written as:

$$I_2^{**} = \left[(n_1 - 1)\frac{\sigma_0^2}{\sigma_1^2}(c_{41} - D\sqrt{1 - c_{41}^2})^2, (n_1 - 1)\frac{\sigma_0^2}{\sigma_1^2}(c_{41} - D_1\sqrt{1 - c_{41}^2})^2 \right] \\ \cup \left[(n_1 - 1)\frac{\sigma_0^2}{\sigma_1^2}(c_{41} + D_1\sqrt{1 - c_{41}^2})^2, (n_1 - 1)\frac{\sigma_0^2}{\sigma_1^2}(c_{41} + D\sqrt{1 - c_{41}^2})^2 \right].$$

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